

## Factor Exposure Analysis

RESEARCH NOTE

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**Factor exposure analysis is a quantitative method that gives investors the ability to measure and understand the return drivers of investment strategies. In general, this type of analysis is grounded in the notion that investment returns are generated through the exposure to different sources of risk premia, or factors<sup>1</sup>. While some risk premia, such as the equity market risk premium, are well known by institutional investors, there are a wide variety of factors that can drive the performance of portfolios over both the short-term and long-term. Additionally, a robust factor exposure analysis allows investors to better judge if active strategies have delivered any “alpha” in excess of their factor exposures.**

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### Measuring factor exposures through linear regressions<sup>2</sup>

The most common approach for measuring factor exposures is through a statistical technique called linear regression. Regressions are very flexible tools that measure the “linear” relationship between a given “dependent” variable, in this case an investment strategy or portfolio, and a collection of “independent” variables, which would be the factors. This analysis is a type of time-series or “return based analysis”, because it is calculated using the historical returns of a given portfolio or strategy, as well as those of the chosen factors<sup>3</sup>.

Based on a linear regression structure, there are some considerations worth highlighting<sup>4</sup>:

#### → **Single-factor vs. multi-factor models, and alpha vs. beta**

A multi-factor model is generally used when conducting a factor exposure analysis. However, within equity markets, the well-known CAPM (Capital Asset Pricing Model) is a single factor specification of the same structure, where the return of the “equity market” is the only relevant factor. In general, utilizing a single-factor model on the returns of a strategy may indicate the presence of “alpha” when, in reality, the difference in returns was achieved through exposures to other factors. That is why including additional factors in a regression (i.e., running a multi-factor model) can provide greater visibility into the driver of returns of strategies, and in many cases, “explain away” portions of what was previously thought of as “alpha”.

### → **The more factors the better? ... Sometimes but not always**

One of the main limitations of CAPM is that, as a single factor model, any source of return that cannot be explained by the market factor will be considered either “alpha” or errors in the model. Both of these issues decrease the effectiveness of CAPM, which is why it would be intuitive to think that adding more factors will lead to better results. However, as the number of factor increases, the robustness of the results are limited by the total sample size, which, in this case, is the number of returns available of the given investment strategy, or “dependent variable”<sup>5</sup>.

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### → **Choosing the right factors and interpreting the results<sup>6</sup>**

As is the case with any quantitative model, its results will only be as useful as the inputs and parameters used to generate them. This is why is very important to choose the right factor, or collection of factors, when doing any factor exposure analysis.

Choosing the right factors can be a qualitative exercise, but in terms of general guidelines, it is worth noting that factors should be selected according to the strategy returns that will be used in regressions. For example, for equity strategies, factors from academic literature (e.g., Fama-French) and/or systematic managers (e.g., AQR), such as Market, Size, Value, Momentum, Quality, and Low Beta, are generally a robust starting point. Fixed Income factors generally include metrics, such as the level of interest rates, the slope of the yield curve, and a measure of credit spreads if the given strategy has some credit exposure.

Other considerations when choosing factors include: investable universe/geography matching (e.g., regressing global strategies on global factors), instability of factors<sup>7</sup> (some factors may be investable while other are not), and others. Even though a model may be well defined in terms of selected factors, the objective is to reduce, as much as possible, any “omitted variable bias”<sup>8</sup>.

### → Single-period vs. rolling-period analysis

Running a regression on the entirety of historical returns available for a strategy provides a good overview of the factor exposures observed over the entire period. However, calculating regressions on rolling periods (e.g., at least 36 months for monthly data) can provide visibility regarding how factor exposures changed through time, something that can be particularly useful for active strategies.

### → Historical (ex-post) vs. prospective (ex-ante) analysis:

Factor exposure analysis is a powerful risk management tool to understand the realized/historical factor exposures of a strategy or portfolio. However, its validity as a forecasting tool is limited. Additional considerations for the usage of factor exposure analysis in forecasting escape the breadth of this paper.

## Factor exposure analysis: illustrative example

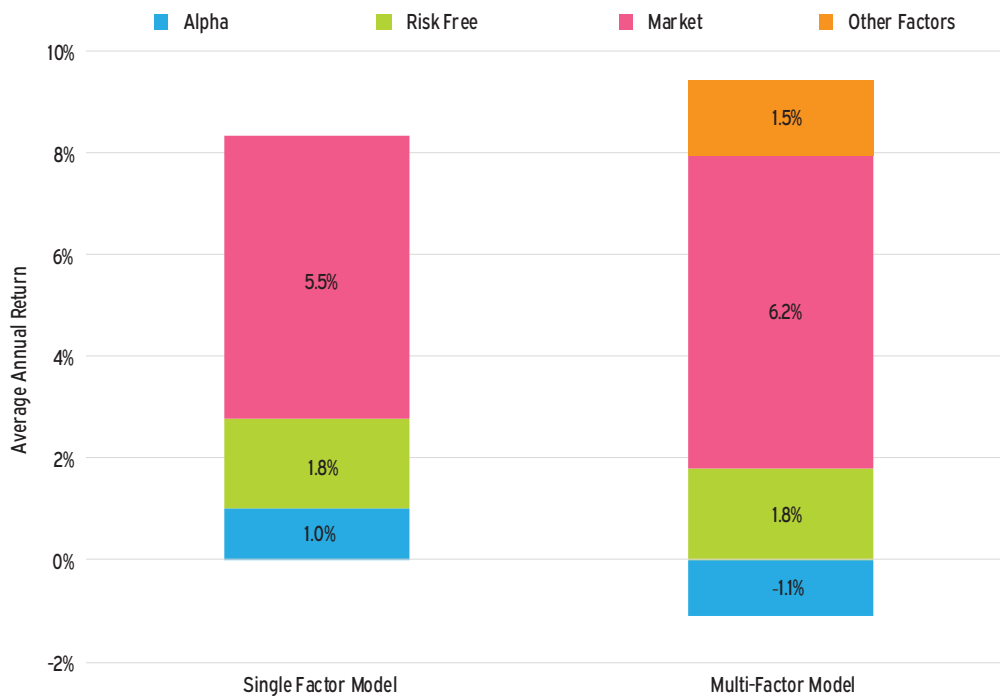
Below is a factor exposure analysis on a simple, long-only domestic equity factor portfolio. The portfolio was constructed as an equal-weighted allocation to MSCI USA's Small Cap, Value Weighted, Minimum Variance, Quality, and Momentum indices, rebalanced monthly<sup>9</sup>. The factors chosen for this analysis were the AQR/Fama-French US factors of Market, Size, Value, Quality, Momentum, and Low Beta ("Betting against Beta")<sup>10</sup>.

	Single Factor Model	Multi-Factor Model
<b>Alpha (annualized)</b>	<b>1.00%</b>	<b>-1.11%</b>
t-stat	1.79	-2.55
<b>Market Beta</b>	<b>0.89</b>	<b>0.99</b>
t-stat	84.91	91.10
<b>Size Beta (SMB)</b>		<b>0.04</b>
t-stat		2.76
<b>Value Beta (HML)</b>		<b>0.07</b>
t-stat		4.87
<b>Quality Beta (QMJ)</b>		<b>0.16</b>
t-stat		8.07
<b>Low Beta Beta (BAB)</b>		<b>0.05</b>
t-stat		4.93
<b>Momentum Beta (UMD)</b>		<b>0.03</b>
t-stat		3.70
<b>R-Squared</b>	<b>0.97</b>	<b>0.98</b>

**TABLE 1**  
**Factor Exposure Analysis.**  
**Regression Results**  
Monthly Log>Returns:  
January 1999 – December  
2019

Table 1 shows that adding factors to the model resulted in realized alpha going from positive to negative, and slightly increased the explanatory power of the model (R-squared increased). However, and perhaps most importantly, the table shows that over the sample period studied, the strategy had positive, and statistically

significant exposure, to all the factors considered, something that would have not been discernible in a single-factor model.



**CHART 1**  
**Return Attribution**  
**through Factor Exposure**  
Monthly Log>Returns:  
January 1999 – December  
2019

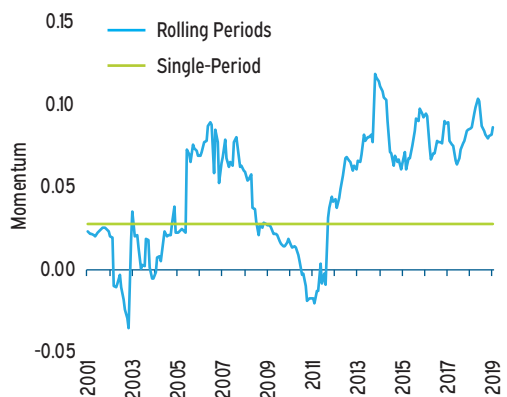
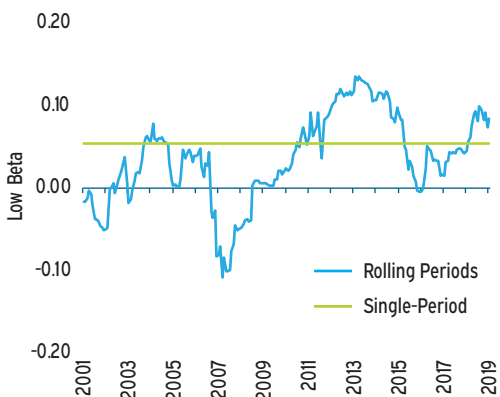
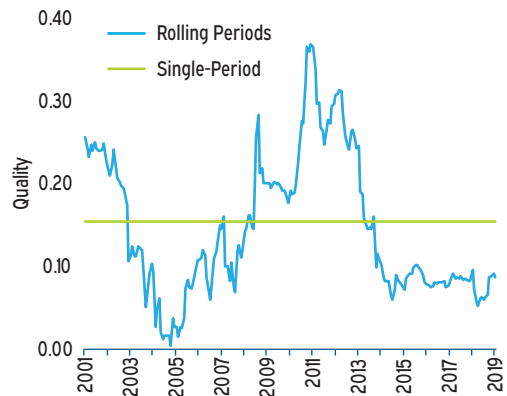
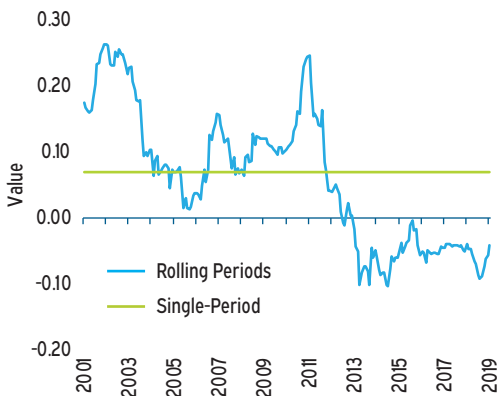
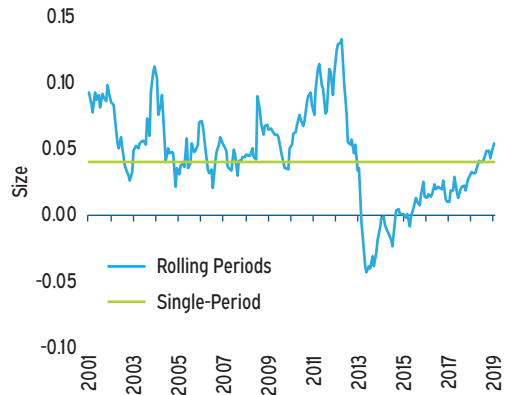
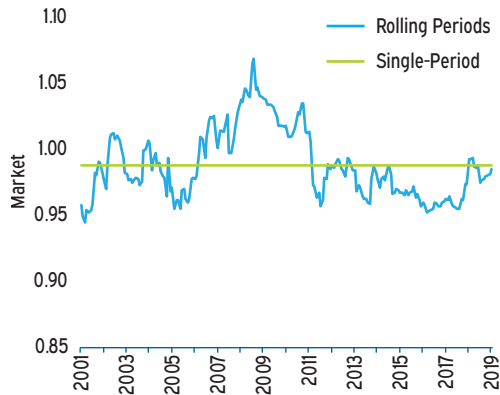
Based on the realized returns of the factors and the portfolio<sup>11</sup>, the factor exposure analysis allows for the calculation of a return-based attribution for the portfolio<sup>12</sup> based on the parameters estimated by the model and the realized returns of the factors. Chart 1 above shows that incorporating additional factors to the model helps attribute ~150 basis points of average annual performance for the strategy, while at the same time, changing alpha attribution from positive to negative.

As is the case with any quantitative model, its results will only be as useful as the inputs and parameters used to generate them. This is why is very important to choose the right factor, or collection of factors, when doing any factor exposure analysis.

While a single-period analysis provided good information regarding the factor exposures of the strategy over the sample period, Chart 2 shows that running a rolling-period analysis provides even greater visibility regarding how these exposures changed through time. From a risk management perspective, it is worth reviewing

these changes through time, as they can provide useful information regarding potential style drift, unintended factor bets, as well as many other, usually difficult to corroborate, investment strategy risks.

**CHART 2**  
**Single-Period vs. Rolling**  
**36-Month Exposures**  
 Monthly Log>Returns:  
 January 1999 – December  
 2019



## Summary

Factor exposure analysis is a powerful tool that allows investors to quantify the factor exposures, or return drivers, of their investment strategies. While the well-known equity market, credit, and interest rate factors are important drivers of returns for strategies of each asset class, both academic and empirical research have discovered multiple additional factors that also drive investment returns<sup>13</sup>.

Well-defined multi-factor models that calculate factor exposures of investment strategies provide investors a deeper understanding of their portfolios, including greater visibility into which factors drive the majority of returns<sup>14</sup>, how exposures change through time (for rolling period analysis), and also, how much “alpha” a given strategy is providing. Incorporating additional factors into a model can “explain away” some, or all, of what a simple single-factor model or excess-return calculation would describe as alpha.

A final word of caution: as is the case with most statistical models, care should be taken when defining the model, selecting the factors, and interpreting the results, in order to achieve a robust and accurate analysis that can be used in the investment decision-making process.

## Appendix 1

### Linear regression specifics

$$R_p - R_f = \alpha + \beta_1 * F_1 + \beta_2 * F_2 + \dots + \beta_n * F_n + \epsilon$$

The equation above provides the general structure of a regression. Through this analysis, the excess returns<sup>15</sup> of a portfolio or strategy “R<sub>p</sub>”, can be explained by the beta exposures “β<sub>n</sub>” of the portfolio to a collection of factors “F<sub>n</sub>”, as well as an intercept term alpha “α”, and an error term “ε”.

Once the regression analysis estimates the parameters of the factor exposure equation, it is important to evaluate both the statistical significance of the parameters of the model (alpha, and betas), as well as the explanatory power of the overall model. The former can be achieved by looking at the t-statistic measure of the parameters, (generally numbers with absolute magnitude above 1.96 are considered statistically significant<sup>16</sup>), while the latter can be evaluated with the R-squared (R<sup>2</sup>) measure, where the higher and closer to “1” the R-squared, the better the factors included explain the variation in returns of the evaluated strategy.

## Appendix 2

### Portfolio and factor returns. January 1999 – December 2019

The portfolio used in the illustrative example was constructed as an equal-weighted allocation to MSCI USA's Small Cap, Value Weighted, Minimum Variance, Quality, and Momentum Indices rebalanced monthly<sup>17</sup>. The factors chosen for this analysis were the AQR/Fama-French US factors of Market, Size, Value, Quality, Momentum, and Low Beta (“Betting against Beta”)<sup>18</sup>

	Average Annual Return	Standard Deviation	Sharpe Ratio
Portfolio	8.3%	13.8%	0.47
Market Factor	6.2%	15.3%	0.29
Size Factor	2.4%	9.4%	0.07
Value Factor	0.4%	9.7%	Neg.
Quality Factor	5.1%	10.0%	0.34
Low Beta Factor	8.1%	14.3%	0.44
Momentum Factor	4.5%	18.2%	0.15
Risk Free Rate	1.8%		

**TABLE 2**  
**Portfolio and Factor Performance**  
Monthly Log>Returns:  
January 1999 – December 2019

## Appendix 3

### Commonly used factors in factor exposure analysis

The list below provides a high-level overview of factors commonly used in factor exposure analysis to explain the performance of assets or investment strategies. However, the construction and definition of each factor can vary by provider, and as such, it can lead to differing results depending on the metric used.

- Equity: Market, Size, Value, Quality, Momentum, Low Beta, etc.
- Fixed Income: Level of interest rates, slope of yield curve, curvature of yield curve, corporate credit spreads, etc.
- Macroeconomic: Inflation, Interest Rates, Trade Weighted Dollar, Systemic Risk, Industrial Production, Retail Sales, Construction Spending, etc.
- Other: Currency Carry, Currency Momentum, Commodities Carry, Commodities Momentum, Commodities Seasonality, etc.

## Appendix 4

### Sources

- Israel R., and Ross, A. "Measuring Factor Exposures: Uses and Abuses", The Journal of Alternative Investments, summer 2017, Volume 20, Number 1.
- Clarke, Roger G and de Silva, Harindra and Thorley, Steven, "Primer on Factor Exposures and Payoffs" (September 29, 2017). Available at SSRN: <https://ssrn.com/abstract=3045449>.
- Benham, Frank and Obregon, Roberto and Walsh, Edmund and Yontar, Timur, "Alternative Beta Strategies" (February 1, 2016). Available at SSRN: <https://ssrn.com/abstract=2726885>

### Footnotes

<sup>1</sup>There is an important distinction to be made between the usage of factors and risk premia. While a factor is any return stream that can "explain" part, or all of the returns of a given strategy, a risk premia is a subset of factors that investors agree should provide positive expected returns over the long term, given their exposure to a given type of risk. For example, equity is not just a factor but a risk premia, because we expect it to provide a positive return over the long term. On the other hand, the returns of oil can be defined as a factor, but it is not necessarily clear that is a risk premia.

<sup>2</sup>As their name suggests, linear regressions measure the linear effects of a set of independent variables on a dependent variable. However, this model is not well-equipped to deal with "non-linear" effects that are sometimes present in finance.



Factor exposures can also be measured using other methodologies such as step-wise regression, ridge regression, vector-autoregression, and even Machine Learning techniques such as elastic net. While they all have technical pros and cons, the details of their use escape the breadth of this paper.

<sup>3</sup>Using generally daily, weekly, or monthly returns.

<sup>4</sup>Appendix 1 for additional details on linear regression equations.

<sup>5</sup>The total number of factors that should be used based on a given sample size of returns is determined by a statistical measure known as “degrees of freedom”. Without going into the specific details, a good rule of thumb should be that the sample size “n” minus the number of factors “k” plus 1, should be at least ~30. Put another way:  $(n - k - 1) \sim \geq 30$

<sup>6</sup>Please refer to the Appendix for an illustrative list of commonly used factors.

<sup>7</sup>In general most factor returns are constructed as indicators, and as such do not include transaction costs of rebalancing if applicable, as well as any potential fee associated with investing in them.

<sup>8</sup>Omitted Variable Bias occurs in statistics when a model leaves out, or does not consider, one or more relevant variables. This results in the model attributing the effects of the omitted variables to the included variables in the model.

<sup>9</sup>See MSCI.com for additional details about the chosen indices. Portfolio created for illustrative purposes only, performance is gross of fees and transaction costs.

<sup>10</sup>See AQR.com for additional details about the chosen factors.

<sup>11</sup>Average annual return for the long-only multi factor portfolio was 8.3% over the studied period. See Appendix 2 for more details on the returns of the portfolio and the factors chosen.

<sup>12</sup>As opposed to the more commonly used “holdings-based” attribution.

<sup>13</sup>See Meketa’s “Alternative Beta Strategies” white paper for additional insights into Equity-market factors.

<sup>14</sup>Technically, R-squared in a regression measures how well the regression model (and its independent variables) explain the “variability” of the dependent variable, or in this case the returns of strategy being analyzed

<sup>15</sup>Excess returns over a risk-free rate “Rf”

<sup>16</sup>95% confidence

<sup>17</sup>See MSCI.com for additional details about the chosen indices. Portfolio created for illustrative purposes only, performance is gross of fees and transaction costs.

<sup>18</sup>See AQR.com for additional details about the chosen factors.

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